1

$$\langle x(\tau)x(0)\rangle = \frac{\langle T^2\rangle}{\gamma^2}\int_{-\infty}^{\tau} dt_1 \int_{-\infty}^{0} dt_2\, e^{-\frac{k}{\gamma}(\tau-t_1)}\, e^{-\frac{k}{\gamma}(0-t_2)}\, e^{-|t_1-t_2|/\tau_c}$$

A

$$e^{\frac{k}{\gamma}t_2}\int_{-\infty}^{t_2} dt_1\, e^{-k/\gamma(\tau-t_1)}\, e^{-(t_2-t_1)/T_c} =$$

$$= e^{k/\gamma\, t_2}\, e^{-k/\gamma\,\tau}\, e^{-t_2/T_c}\int_{-\infty}^{t_2} dt_1\, e^{\left(\frac{k}{\gamma}+\frac{1}{T_c}\right)t_1} =$$

$$= e^{\left(\frac{k}{\gamma}-\frac{1}{T_c}\right)t_2}\, e^{-\frac{k}{\gamma}\tau}\, T_p\left[e^{t_1/T_p}\right]_{-\infty}^{t_2} =$$

$$= e^{\left(\frac{k}{\gamma}-\frac{1}{T_c}\right)t_2}\, e^{-\frac{k}{\gamma}\tau}\, T_p\, e^{t_2\left(\frac{k}{\gamma}+\frac{1}{T_c}\right)} =$$

$$= T_p\, e^{-\frac{k}{\gamma}\tau}\, e^{\frac{2k}{\gamma}t_2}$$

B

$$e^{k/\gamma\, t_2}\int_{t_2}^{\tau} dt_1\, e^{-k/\gamma(\tau-t_1)}\, e^{-(t_1-t_2)/T_c} =$$

$$= e^{k/\gamma\, t_2}\, e^{-\frac{k}{\gamma}\tau}\, e^{t_2/T_c}\int^{\tau} dt_1\, e^{\left(k/\gamma-\frac{1}{T_c}\right)t_1} =$$

$$= e^{\left(\frac{k}{\gamma}+\frac{1}{T_c}\right)t_2}\, e^{-k/\gamma\,\tau}\, T_m\left[e^{t_1/T_m}\right]_{t_2}^{\tau} =$$

$$= e^{\left(\frac{k}{\gamma}+\frac{1}{T_c}\right)t_2}\, e^{-k/\gamma\,\tau}\, T_m\left[e^{\tau/T_m} - e^{t_2/T_m}\right] =$$

$$= T_m\left(e^{-\frac{k}{\gamma}\tau}\, e^{\left(\frac{k}{\gamma}-\frac{1}{T_c}\right)\tau}\, e^{\left(\frac{k}{\gamma}+\frac{1}{T_c}\right)t_2} - e^{-\frac{k}{\gamma}\tau}\, e^{\left(\frac{k}{\gamma}+\frac{1}{T_c}\right)t_2}\, e^{\left(\frac{k}{\gamma}-\frac{1}{T_c}\right)t_2}\right)$$

$$= T_m\left[e^{-\tau/T_c}\, e^{\left(\frac{k}{\gamma}+\frac{1}{T_c}\right)t_2} - e^{-k/\gamma\,\tau}\, e^{\frac{2k}{\gamma}t_2}\right]$$

$$A + B = \underbrace{T_m\, e^{-\tau/T_c}\, e^{\left(\frac{k}{\gamma}+\frac{1}{T_c}\right)t_2}}_{C} + \underbrace{(T_p - T_m)\, e^{-k/\gamma\,\tau}\, e^{2\frac{k}{\gamma}t_2}}_{D}$$

$$\int_{-\infty}^{0} dt_2\, C = T_m\, e^{-\tau/T_c}\, T_p\left[e^{\left(\frac{k}{\gamma}+\frac{1}{T_c}\right)t_2}\right]_{-\infty}^{0} = T_m T_p\, e^{-\tau/T_c}$$

$$\int_{-\infty}^{0} dt_2\, D = (T_p - T_m)\, e^{-\frac{k}{\gamma}\tau}\, T_k\left[e^{2k/\gamma\, t_2}\right]_{-\infty}^{0} =$$

$$= T_k\,(T_p - T_m)\, e^{-\frac{k}{\gamma}\tau}$$

c) $$\langle x(\tau)\,x(0)\rangle = \frac{\langle F^2\rangle}{\gamma^2}\left[T_m T_p\, e^{-\tau/T_c} + T_k\,(T_p - T_m)\, e^{-\frac{k}{\gamma}\tau}\right]$$

$$\langle x(0)\,x(0)\rangle = \frac{\langle F^2\rangle}{\gamma^2}\left[T_m T_p + T_k\,(T_p - T_m)\right]$$

$$\cancel{= \frac{\langle F^2\rangle}{\gamma^2}\left[\left(\frac{k}{\gamma} - \frac{1}{T_c}\right)^{-1}\left(\frac{k}{\gamma} + \frac{1}{T_c}\right)^{-1} + \frac{\gamma}{2k}\left(\frac{k}{\gamma} + \frac{1}{T_c} - \frac{k}{\gamma} + \frac{1}{T_c}\right)\right]}$$

$$\cancel{= \frac{\langle F^2\rangle}{\gamma^2}\left[\frac{1}{k/\gamma - 1/T_c}\;\frac{1}{\frac{k}{\gamma} + \frac{1}{T_c}} + \frac{\gamma}{2k}\,\frac{2}{T_c}\right]}$$

$$\cancel{= \frac{\langle F^2\rangle}{\gamma^2}\left[\frac{\gamma T_c}{kT_c - \gamma}\;\frac{\gamma T_c}{kT_c + \gamma} + \frac{\gamma}{kT_c}\right]}$$

$$= \frac{\langle F^2\rangle}{\gamma^2}\left[\left(\frac{k}{\gamma} - \frac{1}{T_c}\right)^{-1}\left(\frac{k}{\gamma} + \frac{1}{T_c}\right)^{-1} + \frac{\gamma}{2k}\left(\frac{1}{\frac{k}{\gamma} + \frac{1}{T_c}} - \frac{1}{\frac{k}{\gamma} - \frac{1}{T_c}}\right)\right]$$

$$\frac{\frac{k}{\gamma} - \frac{1}{T_c} - \frac{k}{\gamma} - \frac{1}{T_c}}{\left(\frac{k}{\gamma} + \frac{1}{T_c}\right)\left(\frac{k}{\gamma} - \frac{1}{T_c}\right)} = \frac{-\frac{2}{T_c}}{\left(\frac{k}{\gamma}\right)^2 - \frac{1}{T_c^2}}$$

$$= \frac{\langle F^2\rangle}{\gamma^2}\left[\frac{1}{\left(\frac{k}{\gamma}\right)^2 - \frac{1}{T_c^2}} + \frac{\gamma}{2k}\left(-\frac{2}{T_c}\right)\frac{1}{\left(\frac{k}{\gamma}\right)^2 - \frac{1}{T_c^2}}\right] =$$

$$= \frac{\langle F^2\rangle}{\gamma^2\left(\left(\frac{k}{\gamma}\right)^2 - \frac{1}{T_c^2}\right)}\left[1 - \frac{\gamma}{kT_c}\right]$$

(3) $\overline{\Delta x^2(T)} = \langle (x(T) - x(0))^2 \rangle = 2\left(\langle x(0)^2 \rangle - \langle x(T)x(0) \rangle \right)$

$$= 2\left[\frac{\langle F^2 \rangle}{\gamma^2}\left(T_m T_p + T_k (T_p - T_m)\right)\right] - 2\left[\frac{\langle F^2 \rangle}{\gamma^2}\left(T_m T_p e^{-T/T_c} + T_k (T_p - T_m) e^{-\frac{k}{\gamma}T}\right)\right]$$

$$= \frac{2\langle F^2 \rangle}{\gamma^2} T_m T_p \left(1 - e^{-T/T_c}\right) + \frac{2\langle F^2 \rangle}{\gamma^2} T_k (T_p - T_m)\left(1 - e^{-\frac{k}{\gamma}T}\right)$$

$$= 2\frac{\langle F^2 \rangle}{\gamma^2}\left[\frac{1}{\left(\frac{k}{\gamma}\right)^2 - \frac{1}{T_c^2}}\left(1 - e^{-T/T_c}\right) - \frac{\gamma / k T_c}{\left(\frac{k}{\gamma}\right)^2 - \frac{1}{T_c^2}}\left(1 - e^{-\frac{k}{\gamma}T}\right)\right]$$

(I) $T \ll T_c \ll T_k = \frac{\gamma}{2k} < 2T_k = \frac{\gamma}{k}$

$e^{-T/T_c} \approx 1 - \frac{T}{T_c}$

$e^{-\frac{T}{2T_k}} \quad e^{-\frac{T \cdot 2k}{2\gamma}} = e^{-\frac{T}{2T_k}} \approx 1 - \frac{k}{\gamma}T$

$\overline{x^2(T)} \approx \frac{2\langle F^2 \rangle}{\gamma^2 c^2} \frac{T}{T_c}$

~~$\boxed{T_c = 0}$ $\quad \Gamma > 0$~~

~~$$\frac{2\langle F^2 \rangle}{\gamma^2}\left[\frac{1}{\left(\frac{k}{\gamma}\right)^2 - \frac{1}{0^2}}\left(1 - \underbrace{e^{-\infty}}_{\to 0}\right) - \frac{\gamma/k}{T_c\left(\frac{k}{\gamma}\right)^2 - \frac{1}{T_c}}\left(1 - e^{-\frac{k}{\gamma}T}\right)\right]$$~~

$T_c \ll T_k = \frac{\gamma}{2k} < \frac{\gamma}{k}$

$\frac{1}{T_c^2} \gg \frac{1}{(\gamma/k)^2} = \left(\frac{k}{\gamma}\right)^2$

$$\approx \frac{\frac{2\gamma}{kT_c}}{1/T_c^2}\frac{\langle F^2 \rangle}{\gamma^2} = \frac{2\langle F^2 \rangle T_c}{\gamma k} = \frac{\Gamma_F}{k\gamma}$$

$T_c << T_k$

$$\frac{1}{\left(\frac{k}{\gamma}\right)^2 - \frac{1}{T_c^2}} \hat{\approx} -\frac{1}{1/T_c^2} = T_c^2$$

$$\overline{\Delta x^2} \hat{\approx} \frac{2\langle F^2\rangle}{\gamma^2}\left[\underbrace{-T_c^2 \cdot \cancel{(1}}_{\to 0} + \frac{\gamma T_c}{k}\left(1-e^{-\frac{k}{\gamma}\tau}\right)\right]$$

$$\hat{\approx} \frac{2\langle F^2\rangle T_c}{\gamma k}\left(1-e^{-\frac{k}{\gamma}\tau}\right) = \frac{\Gamma_F}{k\gamma}\left(1-e^{-\frac{k}{\gamma}\tau}\right)$$

$T_k << T_c$

$$\frac{1}{\left(\frac{k}{\gamma}\right)^2 - \frac{1}{T_c^2}} \hat{\approx} \left(\frac{\gamma}{k}\right)^2$$

$$\overline{\Delta x^2} \approx \frac{2\langle F^2\rangle}{\gamma^2}\left[\left(\frac{\gamma}{k}\right)^2\left(1-e^{-\tau/T_c}\right) - \underbrace{\frac{\gamma}{kT_c}\frac{\gamma^2}{k^2}}_{\to 0}\right]$$

$$\hat{\approx} \frac{2\langle F^2\rangle}{k^2}\left(1-e^{-\tau/T_c}\right)$$

$\tau << T_c$ $\qquad$ $\cancel{\tau}$ $e^{-\tau/T_c} \hat{\approx} 1-\frac{\tau}{T_c}$

$$\Delta x^2 \hat{\approx} \frac{2\langle F^2\rangle}{k^2 T_c}\tau$$

$$\frac{1}{b^2-a^2}\left[1-\frac{a}{b}+\frac{a}{b}e^{-bt}\right] \qquad \Gamma_F = 2\sigma^2\tau_c$$

$$= \frac{1}{b^2-a^2}\,\frac{a}{b}\left[\frac{b}{a}-1+e^{-bt}\right]$$

$$= \frac{1}{a^2-b^2}\,\frac{a}{b}\left[-\frac{b}{a}+1-e^{-bt}\right) \xrightarrow[a\gg b]{} \frac{1}{a^2}\,\frac{a}{b}\left[1\cancel{\phi}-e^{-bt}\right)$$

$$\cancel{= \frac{a/b}{(a+b)(a-b)}} \qquad = \frac{a}{b}\left[1-e^{-bt}\right)$$

$$\Rightarrow x^2 = \underbrace{\frac{2\sigma^2\cancel{a}}{\gamma^2 b}}\left(1-e^{-t/\tau_c}\right) = 2\sigma^2$$

$$\frac{2\sigma^2\gamma}{\gamma^2\tau_c k} = \frac{2\sigma^2}{\gamma\tau_c k} = 2\sigma^2.$$